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A time-resolved method for nonlinear ultrasonic measurements

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Abstract We describe a time-resolved method for measuring nonlinear ultrasonic phenomena. Current approaches rely on a narrowband measurement of harmonic generation to identify and characterize nonlinearity. Concomitant with these techniques is poor time resolution. We address this limitation with a hybrid narrowband/broadband approach that provides simultaneous time resolution and harmonic isolation for the measurement of weak nonlinearites. We discuss applications and present demonstrative results showing harmonic generation both in water and at a dry contact aluminum-aluminum interface.

Keywords Nonlinear ultrasound, harmonic generation, finite amplitude, nondestructive evaluation, time resolution, contact acoustic nonlinearity

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Introduction Traditional measurement of nonlinear acoustic properties with finite amplitude sound waves involves applying a tone burst to the material and measuring harmonic generation [1]. Because of the use of a burst rather than a pulse and the necessity of distinguishing harmonics from the fundamental, the traditional approach is necessarily narrowband and has poor time resolution. We discuss a more sophisticated approach that builds upon the traditional approach by using a series of tone bursts at different frequencies to make a broadband, time-resolved measurement. A time-resolved measurement allows greatly simplified analysis, including the ability to identify particular wave modes and measure arrival times. It also brings one step closer the practical use of nonlinear measurement for the nondestructive evaluation of cracks and flaws.

Methods Linear systems are usually characterized by measuring their impulse response. The impulse response completely describes any (time-independent) linear system. The impulse response is time-resolved, broadband, and easy to analyze. We will characterize nonlinear media with an analogous approach, by defining the *second harmonic impulse response* to go along with the *linear impulse response*. While we could also define third and higher order impulse responses, we will not discuss them here.

Consider an arbitrary weakly nonlinear system that maps $f(t) \to g(t)$. We can model this mapping with its Taylor Series. For clarity of notation, we will discretize in time (subscripted):

$$g_i \;\approx\; \left.\frac{\partial g_i}{\partial f_j}\right|_{f=0} f_j \;+\; \frac{1}{2!}\left.\frac{\partial^2 g_i}{\partial f_j \partial f_k}\right|_{f=0} f_j f_k \;+\; \frac{1}{3!}\left.\frac{\partial^3 g_i}{\partial f_j \partial f_k \partial f_l}\right|_{f=0} f_j f_k f_l \;+\; ... \quad \begin{pmatrix} \text{sum over} \\ j,k,l \end{pmatrix} \tag{1}$$

A linear (impulse response) model utilizes only the first term of this Taylor expansion. We will use the first term, plus those parts of the second for which $j = k$. Thus, our model $\hat{g}_i$ for g_i is:

$$\hat{g}_i = \left.\frac{\partial g_i}{\partial f_j}\right|_{f=0} f_j \;+\; \frac{1}{2!}\left.\frac{\partial^2 g_i}{\partial f_j^2}\right|_{f=0} f_j^2 \qquad (\text{sum over} j) \tag{2}$$

This model ignores all third and higher order terms, and time-shifted second order self-interaction terms. That is, we ignore nonlinear interactions of the form $f(t)f(t+\Delta)$, and we assume that the nonlinear interaction is of the form $(f(t))^2$.

The term $\left.\frac{\partial g_i}{\partial f_j}\right|_{f=0}$ can be denoted as a constant matrix A_{ij}. Similarly, $\left.\frac{1}{2!}\frac{\partial^2 g_i}{\partial f_j^2}\right|_{f=0}$ can be denoted as B_{ij}. Because our nonlinear system is assumed physically to be time-invariant, these matrices A_{ij} and B_{ij} must have the form of convolutions.

Let us return to continuous time, now denoting our convolutions as **A** and **B**. Our modeled signal can be written as:

$$\hat{g} = \mathbf{A} \otimes f(t) + \mathbf{B} \otimes f^2(t) \tag{3}$$

We will refer to **A** and **B** as the *linear impulse response* and *second harmonic impulse response* respectively.

This model greatly restricts the domain of nonlinear behavior we can analyze. In particular, by using only the first two terms of the Taylor expansion we are limiting ourselves to analyzing only weakly nonlinear phenomena. Furthermore, by ignoring the second order self-interaction terms, we are assuming that the signal never nonlinearly interacts with a time shifted version of itself. Dispersive media, for example, cannot be modelled in this fashion.

The goal of our measurement procedure is to evaluate the convolutions **A** and **B**. We note that the first term of Eq. (3) is linear in $f(t)$, and that the second term is linear in $f^2(t)$. Therefore, in order to determine **A**, we would like to apply an impulse for $f(t)$, and measure the first term of Eq. (3). Likewise, to determine **B**, we would like to apply an impulse for $f^2(t)$ and measure the second term of Eq. (3). We achieve this by applying a series of excitation bursts at different frequencies. The bursts are closely spaced in frequency, so that when linearly superimposed, the composite f is an impulse (Fig. 1). We similarly create a composite impulse f^2 from a series of bursts. In this case, the squared bursts – $cos^2(\omega t)$ – become frequency doubled bursts – $cos(2\omega t)$ – because the DC term is ignored. The terms of Eq. (3) can be isolated from each other by performing narrowband measurements at ω or 2ω. This is achieved by cross-correlating the detected signal with a tone burst at frequency ω or 2ω.

We used a RITEC RAM-5000 system to perform our measurements. This system has high-power gated amplifiers for generating the finite-amplitude tone bursts, and a superheterodyne receiver that can function

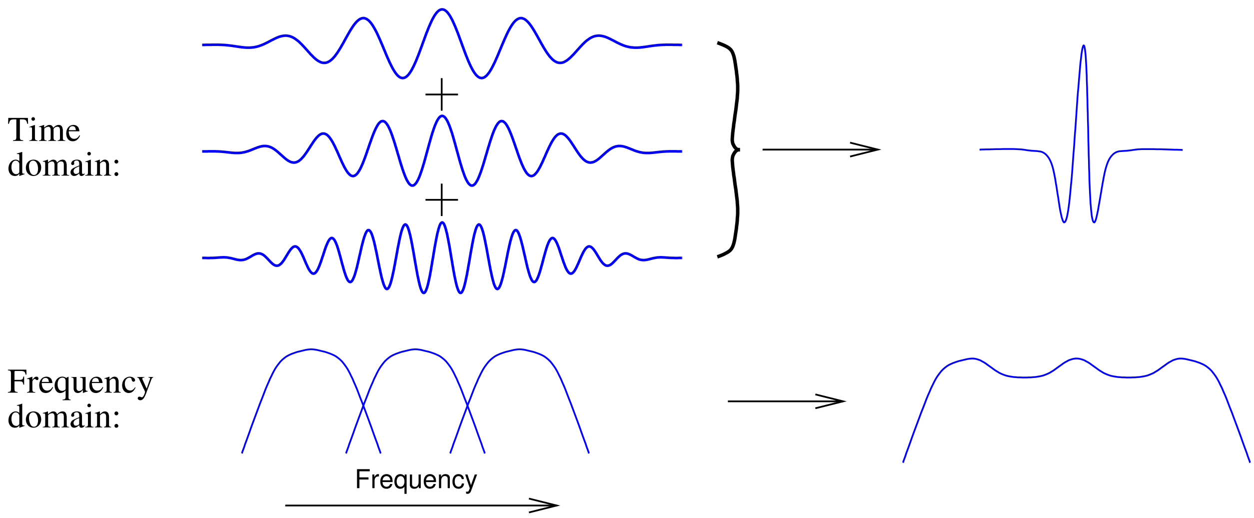


Figure 1: Construction of a broadband impulse from a series of narrowband bursts

as a cross-correlator to measure the response. Our procedure for measuring linear and second harmonic impulse responses follows:

1. **Apply burst**
   Apply a modulated tone burst excitation $s(t) = W_1(t)cos(\omega t)$. (Fig. 2a).

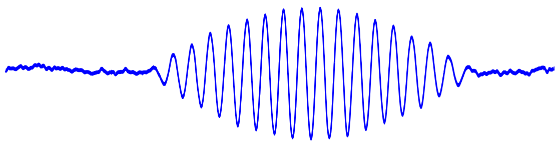
2. **Measure acoustic response**
   The response of the nonlinear system is $\mathbf{A} \otimes s(t)$ plus the frequency-doubled term $\mathbf{B} \otimes s^2(t)$. (Fig. 2b).

3. **Correlate with $W_2cos(n\omega t)$**
   We convolve (correlate in negative time) with a modulated burst $r(t) = W_2(t)cos(-n\omega t)$ to extract the nth (where n=1 or 2) term. (Fig. 2c). The above three steps allow us to probe $\mathbf{A}$ with the burst $s(t) \otimes r(t)$ or $\mathbf{B}$ with the burst $s^2(t) \otimes r(t)$.

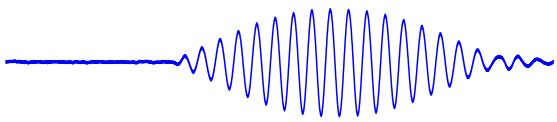
4. **Add the correlated waveforms from bursts at a wide range of frequencies**
   Adding the correlated waveforms (from Fig. 2c) gives our estimate of $\mathbf{A}$ or $\mathbf{B}$. (Fig. 2d).

When these steps are carried out, they provide a measurement of the linear impulse response $\mathbf{A}$ and the second harmonic impulse response $\mathbf{B}$.

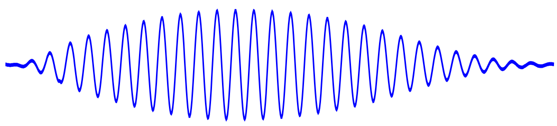
Experiment: Harmonic Generation in Water To test our method, we used it to evaluate harmonic generation in water. We varied the distance between source and receiver immersion transducers in a water bath and measured the linear and second harmonic impulse responses. We used a burst width of $6\mu s$ and a frequency range of 0.86-7.25 MHz, with an average step size of 38.5 kHz (fundamental) and 19.2 kHz (second harmonic). The actual frequency step sizes varied due to dithering. Fig. 3 shows a sample set of waveforms, corresponding to d = 4mm separation. It shows the direct arrival at t = $7\mu s$ and a reverberation between the source and receiver transducers at t = $12\mu s$. Measured second harmonic



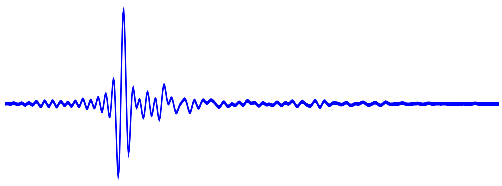
(a) High-power excitation burst



(b) Signal measured by receiver



(c) Received signal correlated with receiver burst



(d) Result from adding correlated bursts from all frequencies

Figure 2: Burst processing demonstration

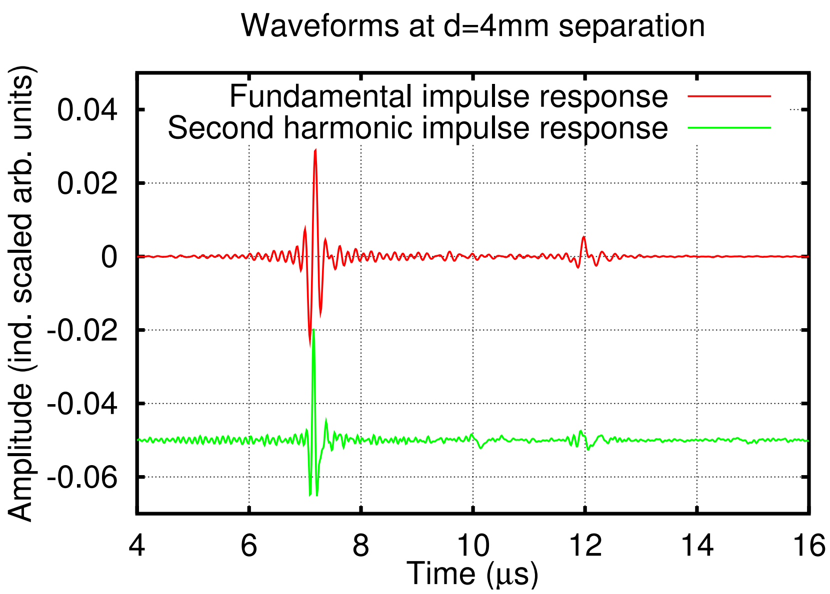


Figure 3: Sample waveforms, harmonic generation in water, propagation distance = 4mm

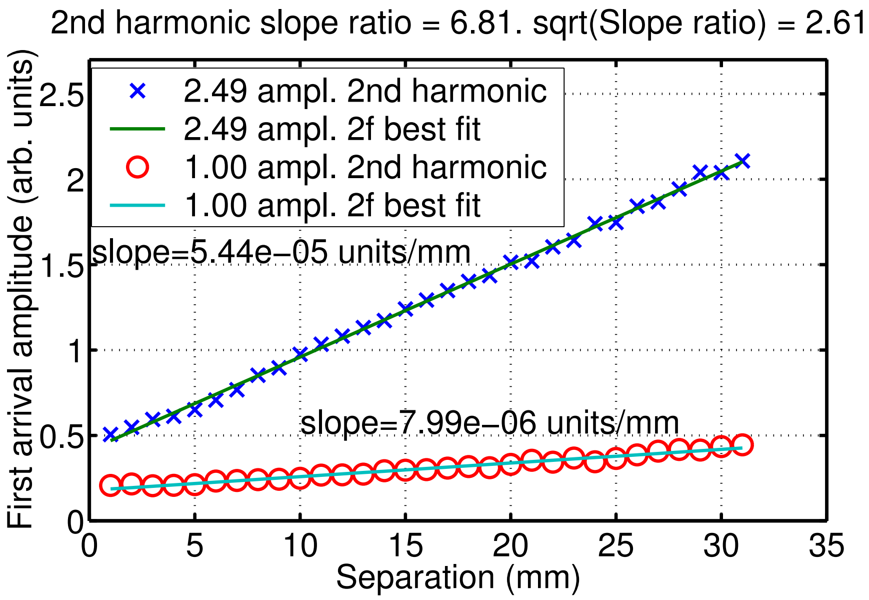


Figure 4: Variation of second harmonic impulse response amplitude with distance of sound propagation through water

amplitudes (peak of the second harmonic impulse response) are shown in Fig. 4 for excitations at relative amplitudes 1.00 and 2.49. The linear impulse response amplitude remained approximately constant over the distance range. The results show that the second harmonic amplitude increases proportionally with distance, as is expected from theory (c.f. [2]). Furthermore, they corroborate that harmonic generation is proportional to the square of excitation amplitude, as $\sqrt{\text{slope ratio}} \approx$ excitation amplitude ratio. These results demonstrate our method for extracting linear and second harmonic impulse responses, and show how our time-resolved method can be used to characterize the nonlinear behavior of water.

Experiment: Harmonic Generation at a Dry Interface In this experiment we evaluated the Contact Acoustic Nonlinearity (CAN) of a dry aluminum-aluminum interface. We placed two bars of aluminum in dry contact over a small area (21.6 mm x 25.4 mm), with the surfaces prepared by grinding. Spring-loaded transducers and honey couplant were used to provide a uniform and maximally linear coupling of ultrasound into the sample. We applied and varied the compressive force normal to the contact area, and measured both linear transmission and second harmonic generation through the interface. A burst width of 4 μs and frequency range of 0.5-5.5 MHz were used with an average step size of 37.3 kHz (fundamental) or 18.6 kHz (second harmonic). Sample waveforms at F=11.4 MPa are shown in Fig. 5. The direct arrival is shown at 14 μs and reverberations in the aluminum bars appear at 22.5 and 26 μs. We measured the linear and second harmonic first arrival amplitudes as the applied load was varied, as shown in Fig. 6. These results demonstrate that the phenomenon we are measuring is nonlinear (as opposed to an artifact of the measurement), since we find the fundamental and second harmonic amplitudes to be non-proportional. According to Solodov [3], we expect the second harmonic amplitude to decrease to zero as we continue to increase loading beyond that shown in Fig. 6.

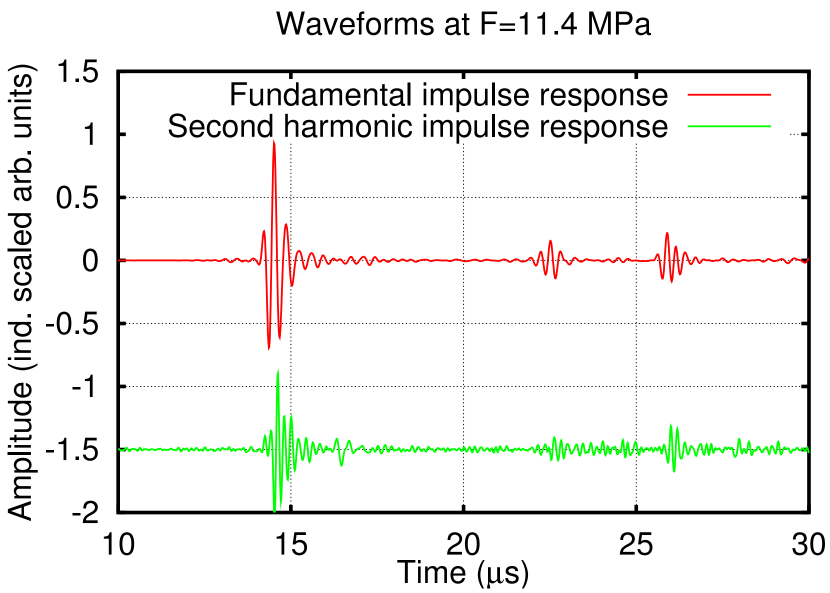


Figure 5: Sample waveforms, Dry-contact Contact Acoustic Nonlinearity (CAN), Applied load/Area=11.4 MPa

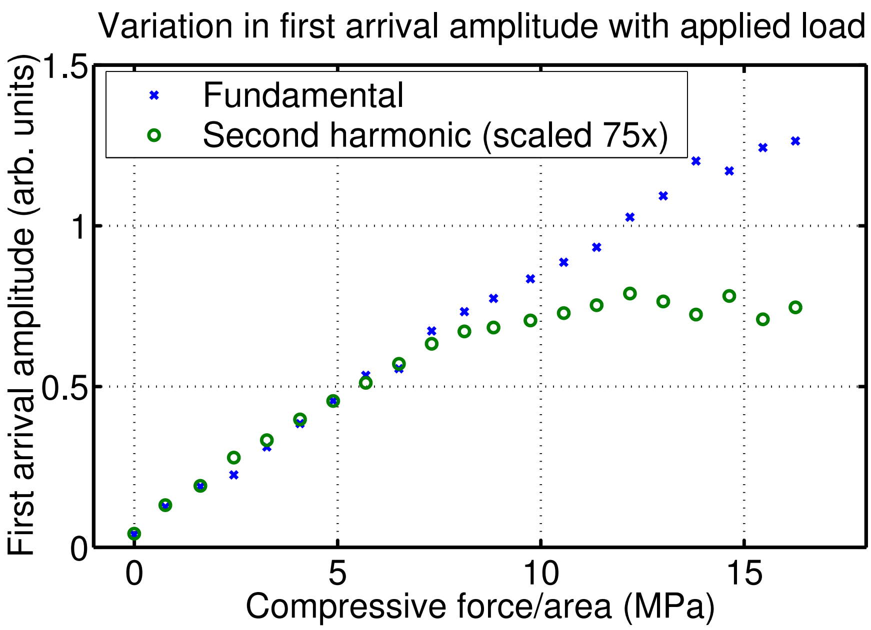


Figure 6: Change of first arrival amplitude with applied compressive load, dry contact CAN

Conclusions We have developed a new method for performing time-resolved measurements of weakly nonlinear acoustic phenomena. The method is built on a mathematical model that allows us to define not only linear, but also second harmonic and higher-order impulse responses. We have constructed and tested a measurement system based on this method and demonstrated its use and utility for two applications. While we have not demonstrated a specific application which *requires* our method, we have shown several applications for which our method provides easier analysis and interpretation than conventional burst or CW measurements.

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