FULL-FIELD VIBRATION MEASUREMENT FOR VIBROTHERMOGRAPHY

Jeremy Renshaw[1] and Stephen D. Holland[2]
[1]Center for NDE and Mat Sci & Eng Dept, Iowa State Univ, Ames, Iowa 50011
[2]Center for NDE and Aerospace Eng Dept, Iowa State Univ, Ames, Iowa 50011

ABSTRACT. Vibrothermography is a nondestructive technique for finding defects through vibration-induced heating imaged with an infrared camera. To model the crack heating process in Vibrothermography, it is essential first to understand the vibration that causes heat generation. We describe a method for calculating internal motions from surface vibrometry measurements. A reciprocity integral and Gauss's law allow representation of internal motion by a surface integral of boundary motion times the Green's Function. We present experimental results showing internal motions calculated from measured surface motions of a vibrating sample. This will ultimately allow estimation of the detectability of a hypothetical crack at an arbitrary location in a specimen.

Keywords: Vibrothermography, Sonic IR, Dynamic Stress, Vibration Measurement
PACS: 43.40.-r, 07.10.Lw, 87.63.Hg, 62.30.+d

INTRODUCTION

We demonstrate a method that measures internal stresses and strains of a vibrating part [1] through surface velocimetry. This allows for a direct calculation of the internal stresses and strains at any point within an object. The mathematics involved in this process are similar to those of the Boundary Element Method (BEM). Understanding the internal vibration around a crack may lead to a relation between the stresses and strains around the crack and the observed heating in Vibrothermography. The ultimate goal of this research is to understand the physics of the crack heating process.

THEORY

The equation of motion governing a solid body is given as [2]

$$\tau_{ij,i} + f_j = \rho \ddot{u}_j \tag{1}$$

with stress tensor τ_{ij}, a body force f_j, density ρ, and displacement u_j. Following the notation used in Achenbach [3], the time-harmonic Green's Function with stress $\tau_{ij:k}^{G:X}$ and displacement $u_{j:k}^{G:X}$ [4] is the solution to Equation 1 given a time harmonic impulse body force in the k'th direction.

CP975, *Review of Quantitative Nondestructive Evaluation Vol. 27,* ed. by D. O. Thompson and D. E. Chimenti

$$\tau^{G:X}_{ij:k,i} + \delta(\underline{x} - \underline{X})e^{-i\omega t}\delta_{jk} = \rho\ddot{u}^{G:X}_{j:k}, \tag{2}$$

The body force is applied at $x = X$ and it is expressed as $f_j = \delta(\underline{x} - \underline{X})e^{-i\omega t}\delta_{jk}$. The reciprocity formulation of Equation 1 is given as Equation 2 multiplied by u_j minus Equation 1 multiplied by $u^{G:X}_{j:k}$.

$$\tau^{G:X}_{ij:k,i}u_j - \tau_{ij,i}u^{G:X}_{j:k} = (\rho\ddot{u}^{G:X}_{j:k} - \delta(\underline{x} - \underline{X})e^{-i\omega t}\delta_{jk})u_j - \rho\ddot{u}_j u^{G:X}_{j:k}. \tag{3}$$

Equation 3 can be simplified due to symmetry and integrated over the specimen volume. Several of the terms in the equation can be grouped and represented as a divergence. Then, these terms can be transformed through the divergence theorem into a surface integral. The remaining terms are the volume integral of a spatial Dirac delta

$$\int_V (\delta(\underline{x} - \underline{X})e^{-i\omega t}\delta_{jk}u_j)dV = -\int_S (\tau^{G:X}_{ij:k}u_j - \tau_{ij}u^{G:X}_{j:k})n_i dA, \tag{4}$$

where n_i is the normal to the part surface at a given location. The surface tractions for this experimental setup are assumed to be zero; therefore, $\tau_{ij}u^{G:X}_{j:k} = 0$ and the equation can be written as

$$\int_V (\delta(\underline{x} - \underline{X})e^{-i\omega t}\delta_{jk}u_j)dV = -\int_S (\tau^{G:X}_{ij:k}u_j)n_i dA, \tag{5}$$

Solving the volume integral thus allows us to calculate the displacement at any given point X as

$$u_k|_{\underline{x}=\underline{X}} = \frac{1}{e^{-i\omega t}}\int_S (-\tau^{G:X}_{ij:k}u_j)n_i dS. \tag{6}$$

Equation 6 shows us that the internal displacement at a point is calculated from a surface integral of the boundary displacement times the Green's Function stress. As the laser vibrometer measures velocity, not displacement, the displacement can be obtained simply through a time integral of the measured velocity. Once the internal displacements are known, the strain can be calculated from the spatial derivative of the displacements. Stress can then be calculated through the use of Hooke's Law. Therefore, the strain and displacement at any point within a part can be calculated from a knowledge of the vector surface velocity of the part. Stress can then be calculated given the material properties of the part in question. A major benefit of this method is that the Green's Function drops off as 1/R in the far field, so the effect of areas that cannot be measured is limited to the immediate vicinity of the missed points.

EXPERIMENTAL PROCEDURE

The vector surface velocity of the specimen is needed to calculate its internal stresses and strains. The vector surface velocity is determined by measuring surface velocity with a laser vibrometer from three linearly independent incident directions. The laser vibrometer measures one-dimensional vibration using the doppler shift in the direction of the beam reflected back from the specimen surface; thus three measurements are needed to obtain the complete vibration of a point. Two in-plane measurements are obtained by rotating the sample by means of a motion control system. A third, out-of-plane measurement is obtained using the motion control system and a carefully-aligned mirror to redirect the beam. The

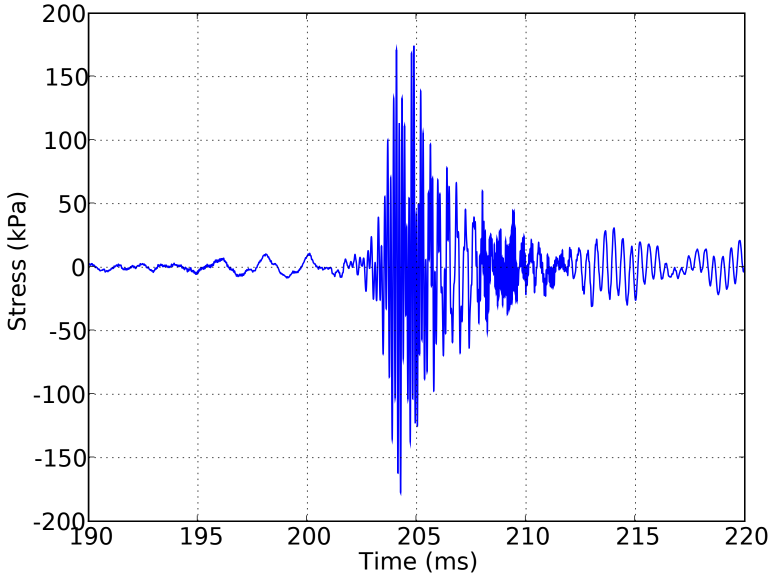

FIGURE 1. Stress as a function of time at a point in the bar using spatial differentiation and Hooke's Law.

motion control system is programmed to scan the measurement beam across the entire surface of the part. As some points - namely those near the transducer - are shadowed and not possible to measure, a few points will always be missed.

A titanium bar 6 inches long, 1 inch wide, and 1/2 inch thick was fastened to a piezoelectric transducer. We covered the bar in retroreflective tape to maximize the reflected optical signal returning to the laser vibrometer. To minimize shadowed areas, the attachment to the transducer was the bar's only means of support. We positioned the bar on a carefully-aligned motion control system and vibrated it using a 100-20000 Hz frequency sweep at low amplitude ensuring repeatable vibration and stresses well within the linear elastic regime. The bar was tesselated into 5mm by 5mm areas and scanned from three selected linearly independent incident directions. The measured data were numerically integrated using Equation 6 to calculate the internal motion of the sample.

RESULTS

The full stress tensor τ_{ij} for any point within the bar can be determined using the derivative of Equation 6 and Hooke's Law. For any frequency, an Inverse Fourier transform gives the stress as a function of time instead of frequency for a given point within the bar. For example, the cross-sectional shearing stress τ_{yz} at a selected point within the test bar is shown in Figure 1. Any component of the stress tensor at any location inside the bar can be accurately calculated and expressed quantitatively, with the exception of the shaded areas near the transducer.

The test bar exhibited several resonances at frequencies within the sweep. One such resonance, a flexural resonance occurring at 7.40kHz, will be examined in more detail. The

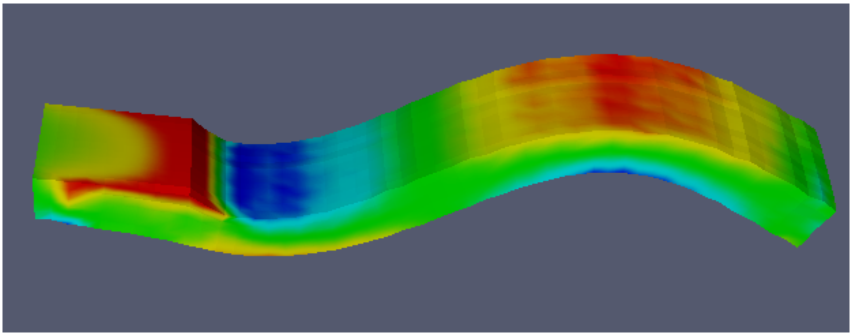

FIGURE 2. 7.4 kHz resonant mode shape measured with the laser vibrometer. Stress τ_{xx} of this mode has been mapped to color.

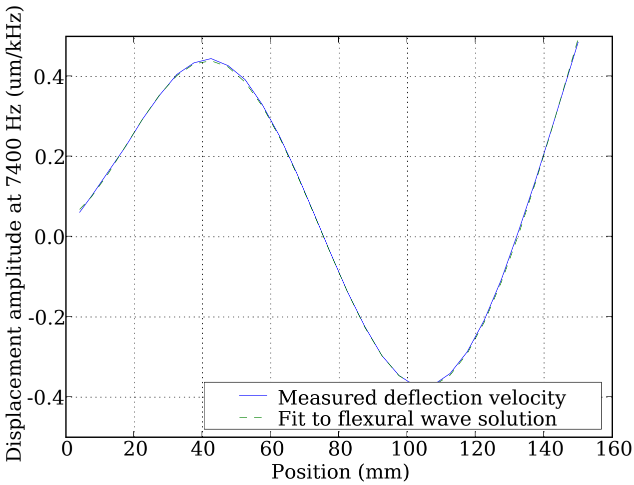

FIGURE 3. Mode profile of the 7.4 kHz resonant mode and the best-fit flexural mode shape. The curve fit is:$y = -.272\sinh(\kappa x) + .272\cosh(\kappa x) + .337\sin(\kappa x) - .226\cos(\kappa x)$ μm/kHz. The two curves are superimposed nearly on top of one another.

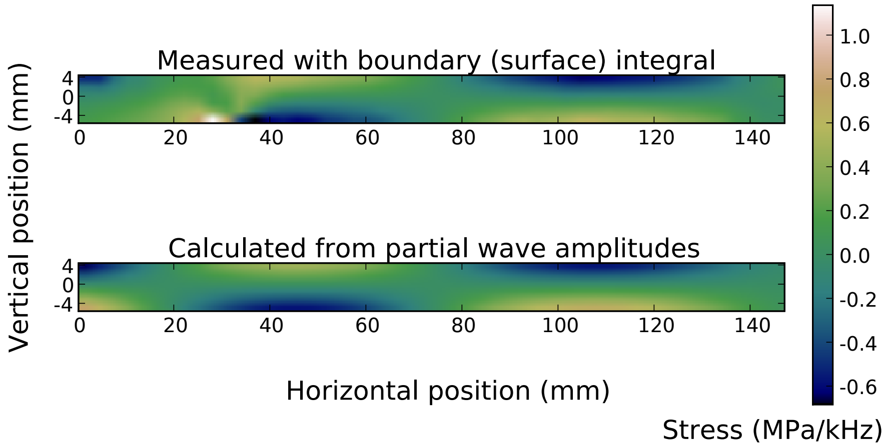

FIGURE 4. Color mapped images of the measured and calculated stresses.

mode shape of this resonance is shown in Figure 2 with the values of stress mapped to color. The region of the bar that appears flat was the area that was shadowed by the transducer. All other parts of the bar appear to follow a flexural wave mode shape and can be fit to the flexural wave solution. Flexural wave theory allows four parameters to fit the displacement profile of a mode. The allowed parameters are coefficients of sin(kx), cos(kx), sinh(kx), and cosh(kx) where k is the wave number and x is the longitudinal position along the bar. These four parameters were fit to the observed shape of the 7.40kHz mode. The velocity profile and the four parameter fit to flexural wave theory are shown in Figure 3.

The four parameters obtained from the best fit to flexural wave theory can be used to evaluate the stress profile in the bar. The τ_{xx} stress is given for the cross-sectional slice of the bar and mapped to color in Figure 4. These values match to within 5% for all areas not shaded by the transducer or its immediate vicinity, corresponding to $x < 35$ mm on the lower left-hand side of the upper bar in Figure 4.

One major advantage of this method for determining stress is shown in Figure 5. This figure shows two missed data points, one on the upper part of the specimen and another near the lower edge. Though these points have a zero velocity, their effects on the surrounding areas quickly drop off as 1/R.

CONCLUSIONS

We have demonstrated a method to calculate the dynamic stress tensor at all points within a vibrating object through the use of surface vibrometry. These results were found to compare to within 5 percent - except in the immediate vicinity of shaded regions - of those calculated based on flexural wave theory using the four parameters shown in Figure 3. The use of the Green's Function and the reciprocity integral formulation allow the measurement to be done even when one or more faces of an object are not measurable, since the influence of missed points or areas dies away with a 1/R dependence from such locations. This method shows promise to accurately correlate internal stresses and strains at a crack face to the observed heating in Vibrothermography.

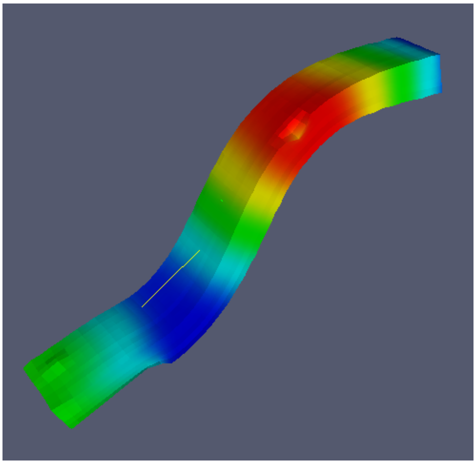

FIGURE 5. Effect of missed data points on surrounding areas. The bar is color mapped to velocity

ACKNOWLEDGEMENTS

This material is based upon work supported by the Air Force Research Laboratory under Contract #FA8650-04-C-5228 at Iowa State University's Center for NDE

REFERENCES

1. C.P. Burger *Handbook on Experimental Mechanics*, edited by A. S. Kobayashi, 260-276, 1987.
2. S. Kobayashi, "Elastodynamics," *Computational Methods in Mechanics* Vol. 3 *Boundary Element Methods in Mechanics*, edited by D. E. Beskos, 1987.
3. J. D. Achenbach, *Reciprocity in Elastodynamics*, 90-94, 2003.
4. N. Kame and T. Yamashita, "Simulation of the spontaneous growth of a dynamic crack without constraints on the crack tip path," *Geophysical Journal International* **139(2)**, 345-358, Nov. 1999.