Measurement of crack opening stresses and crack closure stress profiles from heat generation in vibrating cracks

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(Received 18 June 2008; accepted 1 August 2008; published online 27 August 2008)

A method is described to measure crack opening stresses and closure stress profiles of a surface-breaking crack. Vibration is used to generate frictional heat by rubbing crack face asperities. Heat is generated at regions of contacting crack asperities under low, but nonzero, closure stress. Increasing force is applied to incrementally open the crack and measure the locations of crack heating as a function of applied load. Surface crack closure stresses are approximated from the heating locations as the load is varied and the crack opening stress is measured from the load required to fully open the crack and terminate heat generation. © *2008 American Institute of Physics*. [DOI: 10.1063/1.2976310]

Crack closure, the process by which crack faces contact and carry load,[1] is a controlling factor of crack propagation under loading. A crack in tensile mode I loading must be opened by the crack opening stress, or the stress required to fully open a crack,[2] and have an additional stress applied at the tips to propagate. When the tensile stress is removed, the residual plastic deformed region, formed during crack propagation due to the advancing crack tip, causes the crack faces to contact. This process is known as plasticity-induced crack closure. Closure stresses vary along the crack and can approach the material's yield stress at the crack tips,[1] forcing regions near the crack tips to contact very tightly while other regions far from the tips may remain open. Many crack closure models have been developed since Elber's discovery of crack closure,[2] such as the work of Newman.[3,4] These models describe variations in closure due to crack size, location, geometry, loading, etc.,[5] and extensive experiments and numerical simulations have been done to validate these models on simple cracks,[6,7] however, the difficulty of accurately measuring closure regions has hindered measurements of opening or closure stresses, though ultrasonic methods have shown promise in evaluating closure regions.[8] Compliance techniques[2,9] are more common for measuring opening stresses, but are usually limited to measurements when strain gauges are located near the wake of the crack or crack tip;[9,10] thus experiments may require numerous strain gauges. This paper presents a method to measure surface crack opening stresses and estimate surface closure stress profiles based on thermographic images of closure regions.

This information is deduced from measurements of vibration-induced heat generated at contacting regions (asperities) of a crack. A sample is vibrated, causing asperities under low closure stress to rub and generate heat, which is imaged with an infrared camera (vibrothermography). Open regions of the crack are areas where crack faces do not contact; thus they cannot rub to generate heat. In other areas closer to the crack tips, closure stresses lock asperities together, preventing both rubbing and heat generation. Heat generation sites, therefore, indicate a closure transition region between the open and locked asperities in the crack. Only asperities under low stress (from external loading and closure) generate heat. Changing the external load changes the closure state of a crack and the locations of contacting asperities under low stress. In this manner, crack opening stresses and approximate crack closure stress profiles are measured by tracking locations of low-stress heat-generating asperities as a function of applied external load.

Consider a linear surface crack with a semielliptical depth profile in a bar, as shown in Fig. 1. As the crack grows, it stretches the material around it elastically and plastically. The plastically deformed area around the crack tip forms a permanently stretched plastic zone. In constant amplitude loading, the plastic zone size increases as the crack advances, leaving behind a plastic wake of increasing size,[2] as seen in Fig. 2(a). The plastic wake and plastic zone are referred to as the plastic region and represented by a series of distributed springs whose stiffness is that of the deformed material. The equilibrium length of each spring is related to the amount of permanent stretching, or the width of the plastic region under tensile load, at the spring's location.

When the load growing the crack is released, the material surrounding the plastic wake elastically contracts toward its previous noncracked configuration, compressing the plastic region around the crack. Since the plastically deformed region is larger near the crack tips than at the center of the crack,[2] areas closer to the crack tips are pressed together more tightly than areas further from the crack tips, as seen in Fig. 2(b). Compressing the plastic region creates an unknown closure stress, $\sigma_{xx}^{p}(y,z)$, on the crack faces along the crack length y and depth z. Applying a tensile load reduces

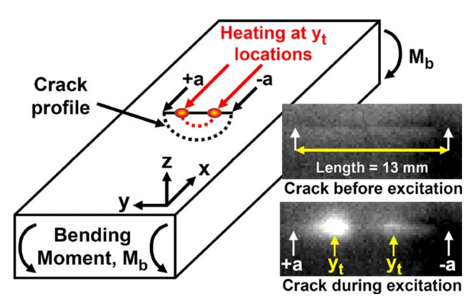

FIG. 1. (Color online) Schematic of a crack in a test bar. The right side shows the IR images of a stationary crack (top) and the IR heating when the crack is vibrated (bottom). Crack tips are located at $\pm a$.

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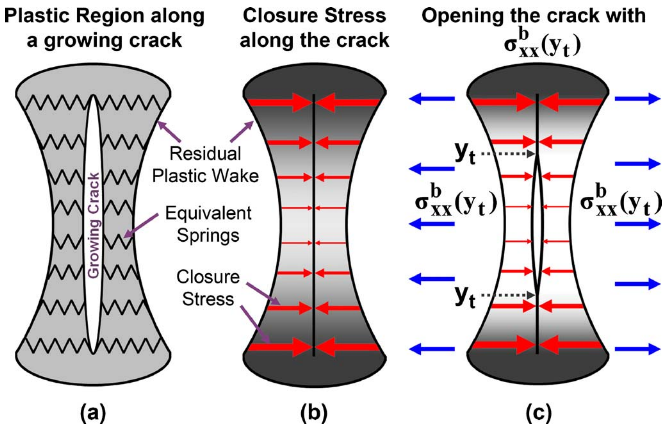

FIG. 2. (Color online) Crack closure model showing (a) the plastic region as the crack advances, (b) closure stresses along a crack due to the compression of the plastic region, and (c) opening the crack with an applied stress to track the closure transition points, y_t, as a function of the applied load.

the compressive closure stresses on the crack faces. Once the closure stress reaches zero at a point, the crack faces at that point separate and act as free surfaces, as shown in Fig. 2(c). Opening a portion of a crack corresponds to decompressing a set of springs until they reach their equilibrium lengths and separate. For simplicity, it is assumed the springs act independently and there is no interaction (i.e., lateral support) between springs.

Locations on the surface ($z=0$) where crack faces separate indicate a transition from closed to open and are referred to as closure transition points, y_t. Since only surface heating can be observed, this method can only obtain information about surface closure states. When a cracked sample is vibrated, asperities surrounding y_t heat up as seen on the right side of Fig. 1 which shows experimental observations of y_t in a partially closed crack. Increasing static tensile stresses are applied in four-point bending to peel open the crack. A separate piezoelectric transducer is placed in contact with the sample to apply low amplitude vibration, generating heat at each closure transition point, y_t. Once the crack fully opens, crack faces no longer rub and no more heat is generated.

Applying a bending moment, M_b, to the bar opens the crack to a corresponding y_t. The stress σ_{xx} on the bar's surface can be approximated by combining terms representing the bending stress, the residual plastic stress, and a possible stress concentration,

$$\sigma_{xx}(y) = \begin{cases} \dfrac{M_b c}{I} + \sigma_{xx}^p(y,0) + \dfrac{K_I^t}{\sqrt{y - y_t}}, & y > y_t \\ 0, & y \leq y_t. \end{cases} \tag{1}$$

The crack at $y > y_t$ is presumed to be closed and at $y \leq y_t$ is presumed open (zero closure stress). The first term of Eq. (1), $M_b c/I$, is the usual expression for bending stress. The second term, $\sigma_{xx}^p(y,0)$, represents the closure stress due to plasticity at y. The third term represents a possible stress concentration around the closure transition point with the normal $1/\sqrt{r}$ dependence. This term is included since the other terms do not represent the stress concentration usually found at the tip of a crack or notch. Our argument, along the lines of Dugdale,[11] is that there cannot be a physical stress concentration around y_t since the stress at y_t must be zero or the crack will incrementally open or close, moving y_t to a different location where the stress would again be zero. Thus, the third term of Eq. (1) must be zero or it would cause large variations in stress over short distances near y_t that are clearly nonphysical given the freedom of the crack to open or close. Equation (1) is approximate because, other than including a stress concentration term, it does not represent the effect of the modified boundary conditions as the crack opens and closes with the applied moment M_b. The bending stress at the crack is slightly higher than shown in Eq. (1) due to the reduction in the cross sectional area as the crack peels open. Higher order terms, possibly needed to fully satisfy the boundary conditions, are also neglected. Solving Eq. (1) for $\sigma_{xx}^p(y_t,0)$ at the closure transition point ($y=y_t$, $\sigma_{xx}=0$), the opening and closure stresses are approximated as

$$\sigma_{xx}^p(y_t,0) = -\frac{M_b c}{I}, \tag{2}$$

where the term on the right, the bending stress, can be easily measured with strain gauges. Equation (2) shows that the crack closure stress at a point y is proportional to the bending stress required to open the crack up to y (move y_t until it meets y). This method, in reality, measures the crack partial-opening stresses, but per the argument above, this gives an approximation of the closure stresses along the crack. Once the crack opening stress is reached, the crack faces completely separate and no further heat is generated. Thus, this method approximates crack closure stress profiles by tracking y_t (heating) locations as a function of the applied stress or load and directly measures the crack opening stress once the crack fully opens and can no longer generate frictional heat.

Linear surface cracks with semielliptical depth profiles were grown in bending in titanium (Ti 6-4) bars using a rectangular EDM starter notch of width 0.75 mm that was subsequently machined off. Figure 1 shows the coordinate system used and the specimen geometry. The cracks were grown in a servohydraulic test machine to a length of about 13.0 mm. The crack in sample A was grown with an R-ratio (min/max stress) of 0.1 and in sample B with an R-ratio of 0.75. The higher R-ratio of sample B was intended to grow a crack of a comparable length to sample A with a higher opening stress[12] due to differences in the plastic region around the crack. Once the final crack length was reached, the samples were removed from the test machine, or given an underload, and strain gauges were applied to the samples to directly measure the applied stresses in the next testing phase. After unloading the samples, both x-ray and eddy current measurements verified that the crack in sample B was a tighter crack than the crack in sample A. Hence, cracks of similar lengths with different closure profiles were compared to observe the effects of closure on heat generation in the cracks. Next, each sample was placed in a four-point bending fixture to control the applied static load and vibrated at low amplitude at its third-order flexural resonance using a piezoelectric transducer. The four-point bending mounts were placed at nodal points to minimize the effect of the mounts on the applied vibration. The samples were vibrated as the applied static load was incrementally varied to measure the change in position of the closure transition regions (y_t) with respect to the crack length as a function of the applied static stress. The combination of applied stress and vibrational

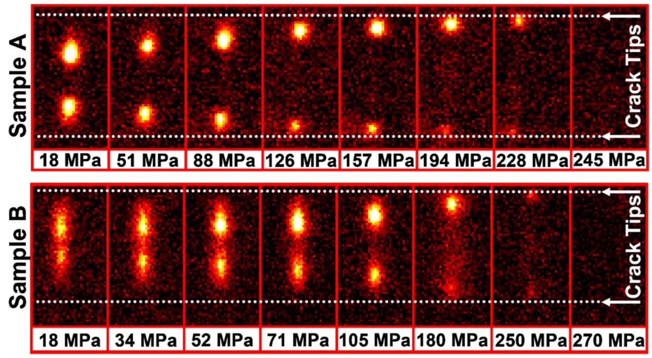

FIG. 3. (Color online) Regions of crack heating as a function of applied bending stress. The far right images show the crack opening stress or the applied stress required to fully open the crack and terminate heat generation.

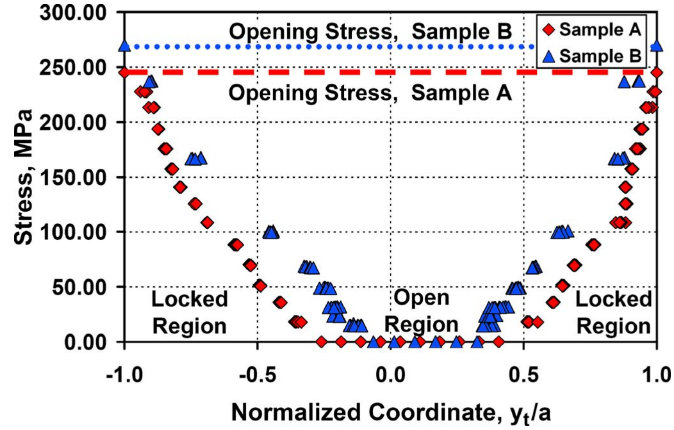

FIG. 4. (Color online) Crack opening stresses and closure stress profiles along the lengths of two cracks calculated from measures of y_t and Eq. (2) and normalized by y_t/a.

stress was carefully controlled to prevent crack growth during the experiment.

Regions of heat generation were used to measure y_t locations as a function of the applied static bending stress. Figure 3 shows heating locations at several applied bending stresses. Note that closure transition points moved toward the crack tips with increasing static tensile stress. The location of the crack tips (at $+a$ and $-a$) are indicated by the white arrows in Fig. 1 and the dotted lines in Fig. 3. Heat generation occured at two y_t regions, both a short distance apart, but not at the crack tips or in the center of the crack. The open region (between the two y_t locations) and the locked asperity regions (near the crack tips) did not generate heat.

Crack opening stresses were measured at the applied static stress required to fully open the crack faces. Once the crack faces were completely separated, they could no longer rub together to generate heat. This is indicated by the two images on the far right of Fig. 3 where no heat generation is observed. The crack opening stress for sample A was 245 MPa and 270 MPa for sample B. Measuring the locations of heat generation in Fig. 3 and applying Eq. (2) gives an approximation of the closure stresses of each crack. Closure stresses versus crack lengths are plotted in Fig. 4. Figure 4 shows that the crack in sample B was tighter and had a higher crack opening stress than the crack in sample A.

This paper presents a method capable of measuring surface crack opening stresses and surface closure stress profiles along a crack using frictional heat generated from vibration-induced rubbing of contacting asperities in a crack. Vibration generates heat at low-stress contacting asperities in a crack from frictional rubbing. Heat generation occurs at locations of low closure stress, referred to as closure transition points, y_t. The closure transition points are used to approximate surface crack closure stress profiles along the entire length of a crack. Opening the cracks to their tips terminated heat generation since the crack faces were no longer in contact and could not rub together to generate frictional heat. Thus, crack opening stresses were measured as the tensile stress required to open the cracks and terminate heat generation. This measurement method offers several advantages over current methods of measuring crack opening and crack closure stresses. It does not require the installation of numerous strain gauges or strain gauge arrays, is not dependent of the proximity of strain gauges to the crack, and gives a direct measurement of the crack opening stress. Also, it can be performed after unloading a crack, as presented in this paper, or *in situ* during fatigue crack propagation.

This material is based on work supported by the Air Force Research Laboratory under Contract No. FA8650-04-C-5228 at Iowa State University's Center for NDE.

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